(DM 03)

M.Sc. (Previous) Mathematics DEGREE EXAMINATION, MAY 2007

Paper III - **COMPLEX ANALYSIS AND SPECIAL FUNCTIONS AND PARTIAL DIFFERENTIAL EQUATIONS**

Time : Three hours Maximum : 100 marks

Answer any FIVE questions choosing atleast TWO questions from each Part.

PART - A

1. (a) (i) Show that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left(x^2 - 1\right)^n$

(ii) Express $P(x) = x^4 + 2x^3 + 2x^2 - x + 3$ in terms of Legendre's polynomials.

(b) Prove that $n P_n(x) = x P'_{n-1}(x)$.

2. (a) Find the solution of the Bessel's differential equation of order *n* of the first kind, *n* being non-negative constant.

(b) (i) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

(ii) Show that $xJ'_n(x) = -n J_n(x) + x J_{n-1}(x)$.

3. (a) Prove that $\int_{-1}^{1} P_m(x) P_n(x)\, dx = 0, \quad if\ m \neq n$

$$= \frac{2}{2n+1}, \ if\ m = n$$

with the usual notation.

(b) Show that

$\cos x = J_0(x) - 2J_2(x) + 2J_4(x) + ...and$

$\sin x = 2J_1(x) - 2J_3(x) + 2J_5(x) + ...$

4. (a) Verify that the equation $yz\, dx + \left(x^2 y - zx\right) dy + \left(x^2 z - xy\right) dz = 0$ is integrable and find its primitive.

(b) Solve $\left(D^2 - 2DD' + D'^2\right) Z = e^{x+2y}$ with the usual notation.

5. (a) Find a complete integral of the equation $2zx - px^2 - 2qxy + pq = 0$ using Chorpits method.

(b) Solve $2x^2 r - 5xys + 2y^2 t + 2(px + qy) = 0$ by Monge's method.

PART B

6. (a) For a given power series $\sum_{n=0}^{\infty} a_n (z-a)^n$, define the number R, $0 \le R \le \infty\, by \frac{1}{R} = \lim\sup |a_n|^{1/n}$. Then prove that (i) if $|z-a| < R$, the series converges absolutely (ii) if $|z-a| > R$, the series diverges (iii) if $0 < r < R$ then the series converges uniformly on $\{z : |z| \le r\}$.

(b) Show that $f(z) = \bar{z}$ is not analytic.

7. (a) Define mobius transformation. Show that a mobius transformation takes circles onto circles.

(b) (i) State and prove the symmetry principle.

(ii) Show that $u(x,y)=e^x \cos y \, \forall (x,y) \in R^2$ is harmonic. Find an analytic function *f* whose real part is *u*.

8. (a) Let γ be a closed polygon : $[1-i, 1+i, -1+i, -1-i, 1-i]$. Then find $\int_\gamma \frac{1}{2} dz$.

(b) State and prove Cauchy's integral formula first version.

9. (a) State and prove the liouvilles theorem.

(b) Suppose that υ is a connected open set and *f(z)* is an analytic function such that $|f(z)| = c$, a constant for $z = \upsilon$. Then show that *f(z)* is a constant.

10 (a) State and prove the Residue theorem.

(b) Evaluate $\int_0^{\pi/2} \frac{d\theta}{a+\sin^2\theta}$ using Residue theorem.